Math 5535 – Homework I

Hand in the starred problems only on Wednesday, September 21

From the text.

Section Problems
9.1 2, 3, 4, *5, *6, 7, 9
9.2 1, 2, 4a, 4b, 4d, *4e, *5
9.3 1, 2, 3, *4, *8, 9, 11, 12, 15, *17, *19

Monday problem sessions

Date Problems
Sept. 12 Section 9.1
Sept. 19 Sections 9.2, 9.3 and additional problems

Plus the following additional problems.

1*. Let \( f(x) = 1 + \tanh(x) = \frac{2}{1+e^{-2x}} \). Prove that \( x \) has a globally attracting fixed point \( \bar{x} \in [1,2] \), i.e., \( \bar{x} \) is a fixed point such that for every initial state \( x_0 \in \mathbb{R} \), we have \( x_n \to \bar{x} \) as \( n \to \infty \).

2*. Suppose \( f: \mathbb{R} \to \mathbb{R} \) is an odd function, that is, \( f(-x) = -f(x) \). Show, first of all that \( \bar{x} = 0 \) is a fixed point. Next, suppose \( x_0 \neq 0 \) is a point such that \( f(x_0) = -x_0 \). Show that \( \{x_0, -x_0\} \) is a periodic orbit of (minimal) period 2. Finally, suppose \( x_0 \neq 0 \) is a point such that \( f^q(x_0) = -x_0 \) and that \( q \) is the least positive number with this property. Show that \( x_0 \) is a periodic point for \( f \) of (minimal) period \( 2q \).

Hints: Section 9.1, problem 6. Prove that each branch of \( T^n \) has a formula of the required form by mathematical induction.

Section 9.2. This would be a good time to use the computer to make some cobweb plots. Then see how much you can prove. For problem 2b. Just do the intervals between the fixed points rigorously and experiment with the other intervals. Understanding the other intervals will be easier using ideas from section 9.3.

Section 9.2., problem 5 uses the intermediate value theorem repeatedly.