1. For any scalar $c$ in $\mathbb{R}$, consider the symmetric matrix $A = \begin{bmatrix} c & 1 & 1 \\ 1 & c & 1 \\ 1 & 1 & c \end{bmatrix}$.

(a) Show that $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is an eigenvector for $A$. What is its eigenvalue?

(b) What are all of the eigenvalues of $A$?
(c) Find an explicit orthonormal basis \((v_1, v_2, v_3)\) for \(\mathbb{R}^3\) consisting of eigenvectors for \(A\).

(d) Find an explicit \(3 \times 3\) matrix \(P\) which is orthogonal \((P^{-1} = P^\top)\) and for which \(P^\top AP\) is diagonal.