Math 3592H Honors Math I
Quiz 3, Thursday Nov. 3, 2016

Instructions:
15 minutes, closed book and notes, no electronic devices.
There are three problems, worth a total of 20 points.

1. (8 points) Parametrize/describe all solutions to the system
   \[ \begin{align*}
   x + y + z + w &= 0 \\
   x + 2y + 3z + 4w &= 1.
   \end{align*} \]

2. (4 points) Prove or disprove:
   Assume two functions \( f, g : \mathbb{R}^n \to \mathbb{R}^n \) are both differentiable on \( \mathbb{R}^n \),
   and satisfy \( f(g(x)) = x \) and \( g(f(x)) = x \) for all \( x \in \mathbb{R}^n \).
   Then for every \( a \) in \( \mathbb{R}^n \), the Jacobian matrix \( Jf(a) \) is invertible.
3. (8 points total)
Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 0 & 3 & c \end{bmatrix}.$$ 

(a) (4 points) Find all values of $c$ that make $A$ invertible

(b) (2 points) For each of the values of $c$ found in part (a), how many solutions are there to the matrix system $Ax = 0$?

(c) (2 points) For each of the values of $c$ found in part (a), how many solutions are there to the matrix system $Ax = e_1$?