1. (8 points total; 4 points each part)

Consider the function $f : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ defined by $f \left( \begin{array}{c} x \\ y \\ z \end{array} \right) = \left( \begin{array}{c} xy \\ z^2 \end{array} \right)$.

(a) Write down the matrix representing $Df(a)$ at $a = \left( \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right)$.

(b) Compute the directional derivative of $f$ at $a = \left( \begin{array}{c} 1 \\ 1 \\ 1 \end{array} \right)$ in the direction of the unit vector $v = \left( \begin{array}{c} 3/5 \\ 4/5 \\ 0 \end{array} \right)$.
2. (8 points total; 4 points each part)
Assume $f : U \to \mathbb{R}^{100}$ is defined on an open subset $U$ of $\mathbb{R}^3$, and differentiable at some point $a \in U$.

(a) What are the dimensions of the Jacobian matrix $Jf(a)$?

(b) On what subset of points $x \in \mathbb{R}^3$ is the derivative $Df(a)(x)$ defined?

3. (4 points total) Prove or disprove:
The function $f : \text{Mat}(n, n) \to \text{Mat}(n, n)$ sending $X \in \text{Mat}(n, n)$ to
$$f(X) = I + 6X - 5X^2$$
is differentiable on all of $\text{Mat}(n, n)$, and at $X = A$, its derivative $Df(A) : \text{Mat}(n, n) \to \text{Mat}(n, n)$ is
$$Df(A)(H) = 6H - 5(AH + HA).$$