Corrections to the Corrected Printing and Paperback Edition of

Olver, P.J., Applications of Lie Groups to Differential Equations,

Last updated: April 10, 2017

*** On page 10, line 6, change
\( \phi \circ \tilde{\phi}^{-1} : \mathbb{R} \to \mathbb{R} \)
to
\( \tilde{\phi}^{-1} \circ \phi : \mathbb{R} \to \mathbb{R} \)

*** On page 19, line 17, change
\( x \in V_0 = \{ x : |x| < \frac{1}{2} \} \)
to
\( x \in V_0 = \{ -1 < x < \frac{1}{3} \} \)

*** On page 36, line 9, change
\( \varepsilon, \theta \in V \)
to
\( (\varepsilon, \theta) \in V \)

*** On page 43, line 23, change
smooth vector fields
to
smooth vector fields

*** On page 52, Example 1.58, change
Lie proved that
to
Lie proved, [4], that

*** On page 67, line 15, change
sort
to
short

*** On page 70, Exercise 1.8, change
in polar coordinates,
to
in polar coordinates with \( r > 0 \),
On page 72, Exercise 1.24(b), change
\( \mathfrak{h} \subset \mathfrak{g} \) has the property

\( \mathfrak{h} \subset \mathfrak{g} \)

to

\( \mathfrak{h} \subset \mathfrak{g} \) is a normal subalgebra (or ideal), meaning that it has the property

On pages 79–81, the ends of the proofs of Proposition 2.6 and Theorem 2.8 that refer back to (1.40) are subtly flawed, in that, under our definition of a local group action, even when \( g = g_1 \cdot g_2 \) with \( g, g_1, g_2 \in G_x \), there is no guarantee that \( g_1 \in G_{g_2 \cdot x} \), i.e., that \( g_1 \cdot (g_2 \cdot x) \) is defined even though \( g \cdot x = (g_1 \cdot g_2) \cdot x \) is defined.

However, a more direct argument, that avoids this difficulty, is to replace the end of the proof of Proposition 2.6 by the following:

\[ \text{... Conversely, if (2.1) holds everywhere, then} \]
\[ \frac{d}{d\varepsilon} \zeta(\exp(\varepsilon \mathbf{v})x) = 0 \]

where defined, and hence \( \zeta(\exp(\varepsilon \mathbf{v})x) = \zeta(x) = c \) is constant for \( \varepsilon \) in the connected interval containing 0 in \( \{ \varepsilon \in \mathbb{R} \mid \exp(\varepsilon \mathbf{v}) \in G_x \} \). Using the fact that the exponential map is a local diffeomorphism from a neighborhood of 0 in \( \mathfrak{g} \) to a neighborhood of \( e \in G_x \), we conclude that \( \zeta(g \cdot x) = c \) for all \( g \) in an open neighborhood of the identity in \( G_x \). Now, set \( \tilde{G}_x = \{ g \in G_x \mid \zeta(g \cdot x) = c \} \). Applying the preceding argument at the point \( g \cdot x \) for any \( g \in \tilde{G}_x \) proves that \( \tilde{G}_x \) is open, while continuity proves that it is closed in \( G_x \). Thus, by connectivity, \( \tilde{G}_x = G_x \), and the result follows.

Similarly, replace the end of the proof of Theorem 2.8 by:

\[ \text{... We have thus shown that if} \ x_0 \text{ is a solution to} \ F(x) = 0, \text{ and} \ \mathbf{v} \text{ is an infinitesimal generator of} \ G, \text{ and} \ \varepsilon \text{ is sufficiently small, then} \ \exp(\varepsilon \mathbf{v})x_0 \text{ is also a solution. As in the proof of Proposition 2.6, one then shows that} \ \tilde{G}_x = \{ g \in G_x \mid F(g \cdot x_0) = 0 \} \text{ is both open and closed in} \ G_x, \text{ and hence, by connectivity,} \ \tilde{G}_x = G_x. \]

• Thanks to Colin James Stockdale Klaus for correspondence on this point.

On page 83, line 14, change

following

to

following

On page 93, line 8, change

function

to

function
On page 106, line -5, the term inside the parentheses should be $\Xi_\varepsilon(x)$. Thus, the equation should read
\[
\sum_j \frac{\partial^2 \Xi_{\varepsilon}^k}{\partial \varepsilon \partial \xi_j} \left( \Xi_\varepsilon(x) \right) \frac{d \Xi_l}{d \varepsilon}(x) = 0,
\]

On page 111, line 3, change $J_{\tilde{f}}(x)$ to $J_{\tilde{f}}(\tilde{x})$.

On page 123, line 5, change $\tau \frac{\partial}{\partial \tau}$ to $\tau \frac{\partial}{\partial t}$.

On page 151, line after (2.110), change normal subalgebra of $g$ to normal subalgebra (ideal) of $g$.

On page 163, line -3, change differential equations to differential equations.

On page 167, line 2, change $(p^k \cdot l)$ to $(p^k \cdot l)$.

On page 167, Definition 2.83, change differential equations to differential equations.

On page 169, line -9, change Exercise 2.32 to Exercise 2.33.

Thanks to Mariano Hermida de La Rica for the preceding four corrections.
*** On page 187, line 8, change
with respect \( y \)
to
with respect to \( y \)

*** On page 187, line 14, change
\[
\frac{\partial \delta}{\partial \hat{x}} + \frac{\partial \delta}{\partial y} \frac{\partial \eta}{\partial x} + \frac{\partial \delta}{\partial v} \frac{\partial \eta}{\partial y} \frac{\partial \eta}{\partial x}
\]
to
\[
\frac{\partial \delta}{\partial \hat{x}} \frac{\partial \hat{x}}{\partial x} + \frac{\partial \delta}{\partial y} \frac{\partial \eta}{\partial x} + \frac{\partial \delta}{\partial v} \frac{\partial \eta}{\partial y} \frac{\partial \eta}{\partial x}
\]

• Thanks to Wen-xiu Ma and his class for catching this and several other errors.

*** On page 202, lines 13–14, change
linear system of ordinary differential equations,
to
linear, constant coefficient system of ordinary differential equations,

*** On page 207, line -2, change
Example 2.64
to
Example 2.44

*** On page 212, on both line 12 and line 13, change
\( M/G \)
to
\( M/G^2 \)

*** On page 214, line 13, change
\[ \tilde{F}(x) = F[\pi(x)] \]
to
\[ F(x) = \tilde{F}[\pi(x)] \]

*** On page 215, line -1, change
\[ \tilde{F}(\pi_1, \ldots, \pi_{m-s}) \]
to
\[ \tilde{F}(\pi_1, \ldots, \pi_{m-s}) \]
** On page 217, line 6, change
\[ F(R, K) = 0 \]
to
\[ \hat{F}(R, K) = 0 \]

** On page 238, Exercise 3.7, change the formula to
\[
R = \left( \frac{t^2 E}{p_0} \right)^{1/5} h \left[ p_0 \left( \frac{t^6}{p_0^3 E^2} \right)^{1/5} \right]
\]

• Thanks to Kameron Decker Harris for spotting this.

** On page 273, line -7, change
\[ \text{tecnique} \]
to
\[ \text{technique} \]

** On page 280, in the table, change
\[
I_x = x D - y A + \frac{1}{2} x u u_t + t M_x
\]
to
\[
I_x = x D + y A + \frac{1}{2} x u u_t + t M_x
\]

• Thanks to Gehrt Hartjen for checking through this table and the table on page 340 in his Mathematics Diplomarbeit in Aachen, 2001.

** On page 290, line -4, change
Exercise 2.33

to
Exercise 2.35

** On page 331, insert minus sign after equals sign in second displayed equation:
\[
D^*_\Delta Q = -q_t - q_{xx} + u q_x
\]

** On page 340, in line 4 of the table, change
\[-y u_{xxx} + x u_{xyy} + u_{xy} \]
to
\[-y u_{xxx} + x u_{xyy} + u_{xy} \]
*** On page 340, in line 5 of the table, change

\[ u_{xx}(yu_{yt} + \frac{1}{2}u_t) - u_{yy}(xu_{xt} + \frac{1}{2}u_t) \]

to

\[ -u_{xx}(yu_{yt} + \frac{1}{2}u_t) + u_{yy}(xu_{xt} + \frac{1}{2}u_t) \]

*** On page 350, lines -8 to -7, change

their Fréchet
to

their Fréchet

*** On page 366, line -7, change

\[ J \setminus I \]
to

\[ I \setminus J \]

- Thanks to Rob Thompson for catching this and several other errors.

*** On page 381, Exercise 3.16a:

The system does not, in fact have a recursion operator, although there is a recursive formula for generating the higher order symmetries. On the other hand, the related system

\[ u_t = u_{xx} + v^2, \quad v_t = v_{xx}, \]


*** On page 381, Exercise 3.16b:

A proof that the Bakirov system has only one generalized symmetry can now be found in: Beukers, F., Sanders, J.A., and Wang, J.P., One symmetry does not imply integrability, J. Diff. Eq. 146 (1998), 251–260.

*** On page 397, equation (6.17), change closing \} to \}:

\[ \{F,H\}(x) = \langle x ; [\nabla F(x), \nabla H(x)] \rangle, \quad x \in g^*, \]  \hspace{1cm} (6.17)

*** On page 420, line -8, change

procedure.
to

procedure.